

On The Detection of Scalar Field Induced Spacetime Torsion

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We argue that the geodesic hypothesis based on autoparallels of the Levi-Civita connection may need refinement in the scalar-tensor theories of gravity. Based on a reformulation of the Brans-Dicke theory in terms of a connection with torsion determined dynamically in terms of the gradient of the Brans-Dicke scalar field, we compute the perihelion shift in the orbit of Mercury on the alternative hypothesis that its worldline is an autoparallel of a connection with torsion. If the Brans-Dicke scalar field couples significantly to matter and test particles move on such worldlines, the current time keeping methods based on the conventional geodesic hypothesis may need refinement.

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Despite continuing efforts to seek a fundamental description of all the basic interactions in Nature, the role of gravitation in this endeavor remains elusive. In part this may be due to our inability to explore the effects of unified descriptions on scales that are accessible to experiment. However, there are compelling suggestions from astrophysical observations that Einstein's original description of gravity may require the inclusion of hitherto undetected fields of either gravitational or matter origin. Low energy effective string theories are replete with unobserved scalar fields and most unified theories of the strong and electroweak interactions predict fields with astrophysical implications. In 1961 Brans and Dicke [1], [3] suggested a modification of Einsteinian gravitation by introducing an additional scalar field with particular gravitational couplings to matter via the spacetime metric. This is arguably the simplest modification and in this note we suggest that efforts to detect such scalar gravitational interactions may have overlooked a possibility that has experimentally detectable implications.

Relativistic gravitation benefits from a formulation in terms of invariant structures on a spacetime manifold. Einstein's theory takes as a fundamental field the spacetime metric tensor and identifies gravitation with the spacetime curvature associated with this metric. The effect of this curvature on matter fields is recognised as due to the *gravitational force* and it is commonly assumed that massive point test particles have spacetime histories that coincide with time-like geodesics associated with the spacetime metric. In terms of the torsion-free connection compatible with this metric, such histories have self-parallel tangent vectors and may be termed Levi-Civita autoparallels. Assuming that our Sun generates an exterior Schwarzschild metric in the vicinity of the planet Mercury, one may use this hypothesis to calculate the perihelion shift per revolution of its orbit and compare directly with observation. Despite competing perturbations, this prediction is regarded as one of the classical tests of any theory of gravitation. Einstein, Infeld and Hoffmann [4] made valiant attempts to prove this *geodesic hypothesis* for particles from a field theory approach but their conclusions were not entirely convincing. However, due to its naturalness, the geodesic hypothesis for spinless test particles has almost become elevated to one of the natural laws of physics [5] and in (pseudo-) Riemannian spacetimes it arises convincingly as the lowest approximation to a multipole expansion of matter distributions in tidal interaction with gravity [6], [7]. For massive test particles with spin and for all matter in spacetimes with torsion, the foundation of this hypothesis is less clear cut [8], [9]. Contrary to popular belief one does not require spinor fields to generate spacetime torsion. If one takes the view that the general setting for any description of gravitation is a manifold equipped with a metric tensor field g and a connection ∇ , then the question of the existence of torsion depends on the connection. Connection forms (gauge potentials) have proved the cornerstone in the unification of the strong and electroweak interactions and it is natural to treat them as independent dynamical entities in gravitation also. The dynamical status of spacetime torsion is then dependent on the action chosen for the theory with independent metric and connection fields.

For any vector fields X and Y on spacetime the torsion tensor \mathbf{T} is defined by:

$$\mathbf{T}(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y].$$

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In spacetimes with a (pseudo-)Riemannian geometry one assumes $\mathbf{T} = 0$ and $\nabla \mathbf{g} = 0$ and the connection is given in terms of the Christoffel symbols $\{\Gamma_{\nu\lambda}^{\mu}\}$ obtained by differentiating components of the metric tensor. To distinguish this Levi-Civita connection from a general one we denote it $\hat{\nabla}$.

In the scalar-tensor theory of gravitation formulated by Brans and Dicke [1], the motion of a test particle was originally assumed to be a Levi-Civita geodesic associated with the metric derived from the Brans-Dicke field equations (even though the scalar field could vary in spacetime). Later Dirac in [2] showed that in a Weyl invariant generalisation it was more natural to generate the motion of a test particle from a Weyl invariant action principle and that such a motion in general differed from a Brans-Dicke Levi-Civita geodesic. Although Dirac was concerned with the identification of electromagnetism with aspects of Weyl geometry even for neutral test particles it turns out that test particles would follow auto-parallels of a connection with torsion. In Ref. [10] we have shown that the theory of Brans and Dicke [1] can be reformulated as a field theory on a spacetime with dynamic torsion determined by the gradient of the Brans-Dicke scalar field Φ . Of course no new physics *of the fields* can arise from such a reformulation, although it does clarify certain issues relating to the conformal structure of the theory and its couplings to matter with intrinsic spin. However, the behaviour of spinless *particles* in such a geometry with torsion depends on the choice made from two possible alternatives. One may assert that their histories are *either* geodesics associated with autoparallels of the Levi-Civita connection *or* the autoparallels of the non-Riemannian connection with torsion. Since one may find a spherically symmetric, static solution to the Brans-Dicke theory (in either formulation), it is possible to compare these alternatives for the history of Mercury about the Sun by regarding it as a spinless test particle as in General Relativity. In this letter we report on the results of this computation.

In terms of local coordinates $\{t, \rho, \theta, \phi\}$ the spherically symmetric static solution [1] in the vicinity of Mercury is taken in the form:

$$\mathbf{g} = - \left(\frac{1 - \frac{\Lambda}{\rho}}{1 + \frac{\Lambda}{\rho}} \right)^{2p} c^2 dt \otimes dt + \left(1 + \frac{\Lambda}{\rho} \right)^4 \left(\frac{1 - \frac{\Lambda}{\rho}}{1 + \frac{\Lambda}{\rho}} \right)^{2-2p-2q} (d\rho \otimes d\rho + \rho^2 d\theta \otimes d\theta + \rho^2 \sin^2 \theta d\phi \otimes d\phi)$$

$$\Phi = \Phi_0 \left(\frac{1 - \frac{\Lambda}{\rho}}{1 + \frac{\Lambda}{\rho}} \right)^q$$

where, in terms of the conventional Brans-Dicke coupling parameter ω , $p = \left(\frac{2\omega+4}{2\omega+3} \right)^{1/2}$, $q = p(\gamma - 1)$ with $\gamma = \frac{\omega+1}{\omega+2}$. The constants Λ and Φ_0 are fixed by ensuring that in the weak field limit [1] one can identify the Newtonian coupling constant G and the source of matter as the solar mass M :

$$\Lambda = \frac{GM}{2c^2} \left(\frac{2\omega+3}{2\omega+4} \right)^{1/2}$$

$$\Phi_0 = \frac{1}{G} \frac{2\omega+4}{2\omega+3}.$$

In order to relate this solution to a frame more appropriate to terrestrial observation, we effect the transformation

$$r = \rho \left(1 + \frac{\Lambda}{\rho} \right)^{1+p+q} \left(1 - \frac{\Lambda}{\rho} \right)^{1-p-q}$$

and express the metric as

$$\mathbf{g} = -A(r)c^2 dt \otimes dt + B(r)^{-1} dr \otimes dr + r^2 (d\theta \otimes d\theta + \sin^2 \theta d\phi \otimes d\phi) \quad (1)$$

where

$$A(r) = 1 - \frac{2GM}{c^2 r} + a_2 \frac{(2GM)^2}{c^4 r^2} + \dots$$

$$B(r) = 1 + b_1 \frac{2GM}{c^2 r} + \dots$$

$$\Phi(r) = \Phi_0 \left(1 + c_1 \frac{2GM}{c^2 r} + \dots \right)$$

with $a_2 = c_1 = \frac{1-\gamma}{2}$, $b_1 = -\gamma$. In the geometry with torsion, the torsion 2-forms defined by $T^a(X, Y) = \frac{1}{2}e^a(\mathbf{T}(X, Y))$ with $\mathbf{g} = -e^0 \otimes e^0 + \sum_{j=1}^3 e^j \otimes e^j$ are given by [10]:

$$T^a = e^a \wedge \frac{d\Phi}{2\Phi}.$$

The equation for a timelike autoparallel is

$$\nabla_{\dot{C}} \dot{C} = 0$$

where the 4-velocity \dot{C} is normalised with

$$\mathbf{g}(\dot{C}, \dot{C}) = -c^2. \quad (2)$$

By expressing ∇ in terms of the Levi-Civita connection $\hat{\nabla}$ with $\tilde{V} = g(V, -)$ for any vector V one may write this as

$$\widetilde{\hat{\nabla}_{\dot{C}} \dot{C}} = -\frac{1}{2\Phi} i_{\dot{C}}(d\Phi \wedge \tilde{C})$$

(the operator $i_{\dot{C}}$ denotes contraction of the 2-form with the vector \dot{C}) and interpret the right hand side as a torsion acceleration field analogous to the Lorentz force on electrically charged particles. Note however that the torsion force produces the same acceleration on all massive test particles. If \dot{C} is parameterised in terms of proper time τ in any coordinates $x^\mu(\tau)$, the above is:

$$\frac{d}{d\tau} \left(\Phi^{1/2} \frac{dx^\mu}{d\tau} \right) + \Phi^{1/2} \{^\mu_{\nu\lambda}\} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = -g^{\mu\nu} \frac{\partial_\nu \Phi}{2\Phi^{1/2}}. \quad (3)$$

(This equation coincides with equation (8.8) in [2] for a neutral test particle in Dirac's reformulation of Weyl's theory mentioned above. The apparent sign difference is due to signature conventions.)) To solve this equation one notes that for any Killing vector K , ($\mathcal{L}_K \mathbf{g} = 0$) with $K \Phi = 0$, expressions proportional to $\Phi^{1/2} \mathbf{g}(K, \dot{C})$ are constant along the test particle worldline. For the above solution (1), the Killing vectors $K_t = \partial_t$ and $K_\phi = \partial_\phi$ generate two proper time constants of the motion E and L corresponding to energy and angular momentum about any direction in space, respectively. Choosing solutions with $\theta = \pi/2$, we set

$$E = -mc \left(\frac{\Phi}{\Phi_0} \right)^{1/2} A(r) \dot{t} \quad (4)$$

$$L = m \left(\frac{\Phi}{\Phi_0} \right)^{1/2} r^2 \dot{\phi}. \quad (5)$$

To facilitate comparison with non-relativistic Kepler orbits we write $E^2 = (mc^2 + \mathcal{E})^2$ in terms of the (constant) rest mass m of Mercury and identify \mathcal{E} with its (negative) gravitational binding energy. Similarly let $L^2 = m^2 h^2$ with $h = \sqrt{GM r_0}$. Then the standard Newtonian Kepler orbit arises from $\Phi = \Phi_0$ above and is given in the non-relativistic limit by the geodesic hypothesis as

$$\frac{1}{r} = \frac{1}{r_0} (1 + \epsilon \cos \phi)$$

where the eccentricity for a bound orbit is $\epsilon = \sqrt{1 + \frac{2\mathcal{E}r_0}{GMm}}$.

From (2) one may eliminate \dot{t} and write the equation for the planar orbit of Mercury according to (3). Taking into account that the speed of Mercury is non-relativistic and that its Newtonian orbit is much larger than the Schwarzschild radius $r_s = 2GM/c^2$ of the Sun, one finds in terms of $U(\phi) = 1/r(\phi)$ the orbit equation

$$\left(\frac{dU}{d\phi} \right)^2 \simeq \frac{2\mathcal{E}}{GMmr_0} + \left(\frac{2}{r_0} - \frac{1}{r_0(\omega + 2)} \right) U - U^2$$

$$+ \frac{(1 - a_2 + b_1 - b_1 c_1)}{L^2} \frac{(2GMm)^2 U^2}{c^4} - \frac{2GMb_1 U^3}{c^2}. \quad (6)$$

We note that the last two terms on the right hand side of this equation yield the leading corrections to the non-relativistic Newtonian orbit. The latter however also receives contributions from the Brans-Dicke interaction. The Newtonian orbit is the Keplerian ellipse:

$$\hat{U}(\phi) = \frac{1}{\hat{r}_0} (1 + \hat{\epsilon} \cos \phi)$$

with

$$\hat{r}_0 = r_0 \frac{2\omega + 4}{2\omega + 3}$$

$$\hat{\epsilon}^2 = 1 + \frac{8\mathcal{E}r_0}{GMm} \left(\frac{\omega + 2}{2\omega + 3} \right)^2.$$

Equation (6) can be integrated exactly in terms of Jacobi elliptic functions.

A straightforward analysis of the periodicity of such solutions enables one to compute the perihelion shift per revolution Δ of the orbit:

$$\Delta = \frac{3\omega + 5}{3\omega + 6} \delta_\omega \quad (7)$$

where $\delta_\omega = 3\lambda_\omega\pi$ and $\lambda_\omega = r_s/\hat{r}_0$. Using the Kepler period $\hat{T} = 2\pi(\hat{r}_0^3/(1 - \hat{\epsilon}^2)GM)^{1/2}$, the shift δ_ω may be expressed in terms of \hat{T} and $\hat{\epsilon}$. This may be compared with the result based on the assumption that Mercury's orbit follows from a geodesic of the torsion-free Levi-Civita connection:

$$\hat{\nabla}_{\dot{C}} \dot{C} = 0.$$

In this case one finds

$$\Delta = \frac{3\omega + 4}{3\omega + 6} \delta \quad (8)$$

where $\delta = 3\lambda\pi$ with $\lambda = r_s/r_0'$ is the perihelion shift per revolution of the orbit based on the Schwarzschild solution for the metric in General Relativity [11].

Constraints on ω imposed by Observation of the Planet Mercury

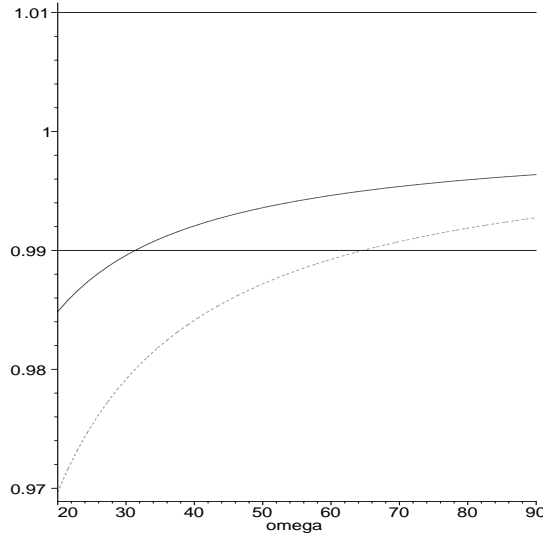


FIG. 1. Behaviour of Δ/δ as a function of ω . The full curve corresponds to a precession rate of Mercury's orbit under both metric and torsional acceleration. The dotted line corresponds to the original prediction of the Brans-Dicke theory. The experimental observations are consistent with Δ/δ lying between the full horizontal lines centered on $\Delta/\delta = 1$

It should be noted that r'_0 is the parameter of a Newtonian ellipse with Kepler period $T = 2\pi(r'_0)^3/(1 - \epsilon^2)GM)^{1/2}$ and eccentricity ϵ . However in terms of the energy and angular momentum of the particle in orbit the relations (4) and (5) are replaced by similar ones but with $\Phi = \Phi_0$.

In order to compare the prediction (7) with that of Brans-Dicke (8), we recognise that the orbit parameters must be common (i.e. constants of motion chosen so that $\hat{r}_0 = r'_0$). Thus with a common δ we may plot Δ/δ as a function of ω in a domain where both enter the error corridor of this value determined by observation. According to [11] the above corridor can probably be reduced by half although we shall be conservative in our error estimates. The differences indicated in the figure suggests that in assessing the significance of the scalar field in the Brans-Dicke description of gravity, one should take seriously the possibility that Mercury's orbit might be described by an autoparallel of the natural connection with torsion used in the alternative formulation of the theory. Given the recent improvements in satellite technology and space location techniques, a more reliable method to detect the effects of torsion induced motion would be to measure the precession of satellites in highly eccentric orbits about spherical asteroids. Indeed, if the Brans-Dicke scalar field couples significantly to matter and test particles move on autoparallels of connections other than the one in General Relativity, current time keeping methods based on the conventional geodesic hypothesis may need refinement.

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- [1] C H Brans, R Dicke, Phys.Rev. **124** 925 (1961)
 - [2] P A M Dirac, Proc. Roy. Soc **A333** 403 (1973)
 - [3] C H Brans *Gravity and the tenacious scalar field*, gr-qc/9705069
 - [4] A Einstein, L Infeld, B Hoffmann, Ann. Math. **39** 65 (1938)
 - [5] S Weinberg, **Gravitation and Cosmology** (Wiley, 1972)
 - [6] A Papapetrou, Proc. Roy. Soc. **A209** 248 (1951)
 - [7] W G Dixon, Proc. Roy. Soc. **A314** 499 (1970)
 - [8] V N Ponomarev, Bull. Acad. Polon. Sci **XIX** 545 (1971)
 - [9] W Adamowicz, A Trautman, Bull. Acad. Polon. Sci **XXIII** 339 (1975)
 - [10] T Dereli, R W Tucker, Phys. Letts **110** 206 (1982)
 - [11] C M Will, **Theory and Experiment in Gravitational Physics** (Cambridge U P , 1993)